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Article (Accepted Version)

Manzini, Paola and Mariotti, Marco (2008) Consumer choice and revealed bounded rationality. Economic Theory, 41 (3). pp. 379-392. ISSN 0938-2259

Economic Theory manuscript No.
(will be inserted by the editor)

Consumer choice and revealed bounded rationality

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Received: date / Accepted: date

Abstract We study two boundedly rational procedures in consumer behavior. We show that these procedures can be detected by conditions on observable demand data of the same type as standard revealed preference axioms. This provides the basis for a non-parametric analysis of boundedly rational consumer behavior mirroring the classical one for utility maximization.

Keywords bounded rationality · revealed preference · consumer choice

PACS D0 · D11

1 Introduction

The original idea of Samuelson [18]'s revealed preference approach was to find conditions under which observed consumer behavior 'reveals' full rationality, in the form of utility maximization. Because it permits direct, nonparametric tests of the theory, this approach enjoys considerable success[1]. But suppose that a consumer does not reveal full rationality. Can he 'reveal' some form of bounded rationality, and if so, how? The question is pertinent in view of the increasing interest in boundedly rational models of individual choice behavior. In this paper we study two forms of bounded rationality that are amenable to tests of exactly the same nature as those used in the revealed preference literature[2].

Our approach is underpinned by the observation that consumer behavior may be stable, consistent, and traceable to simple rules (and therefore predictable in principle), even when it is not determined by utility maximization. For a quick example (more detailed ones are studied later), suppose you observe the choice of a consumer deciding how to allocate his

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[1] See Afriat [1], Houthakker [10], Richter [16] Suzumura [19] and Varian [21] for classic treatments and Varian [22] and Blundell [2] for recent discussions of the modern economic and econometric theory of revealed preference.

[2] In the concluding section we expand on the meaning of 'the same nature'.

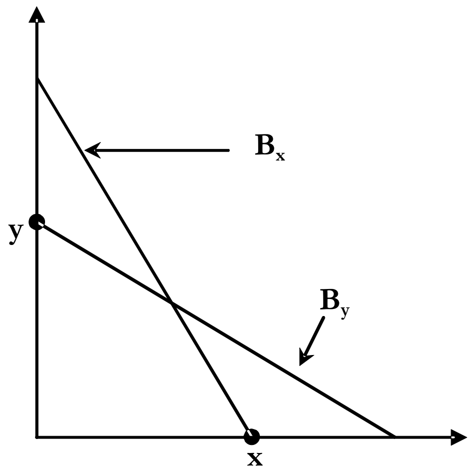

Fig. 1 The status symbol seeker.

budget between platinum and diamonds. Your data consists of two budgets, B_x and B_y, at which the observed choices are x and y, respectively (see figure 1).

These data are inconsistent with the existence of a nonsatiated utility function that the consumer is maximizing: if $u(x) \geq u(y)$, then it cannot be that y is chosen out of B_y, where x costs less. However an observer might 'rationalize' the consumer choice by the following rule of behavior: 'I only care about the most expensive item, on which I spend my entire budget'. Though not fully rational, this demand behavior is as consistent and as predictable as fully rational demand behavior. The data are incompatible with a model of *utility* maximization, but it can be shown that they are still consistent with a model of *maximizing* behavior.[3] As we shall explain in detail below, they are in particular compatible with the (possibly sequential) maximization of *binary relations*: if we knew the binary relation(s), we would know how the consumer would choose in any given situation. The question is, when are the choice data consistent with this type of explanation?

We consider two rather different types of bounded rationality:

1) Retain the assumption of maximizing behavior, but drop all the assumptions that lead from the maximization of a relation to the maximization of utility. The consumer discards all bundles which are dominated according to a strict binary relation, without this relation necessarily exhibiting standard properties such as transitivity or completeness.

2) The consumer uses a sequential procedure, based on *two* binary relations, to first discard consumption bundles and then select from the resulting 'shortlist'.

The idea of 'revelation' of bounded rationality in (1) is not new at the formal level - although it was originally not presented as such. We report an existing (but maybe not widely known) characterization result (theorem 3) due to Kim and Richter [12], which builds on the classical work on revealed preference by Richter [16]. In addition, we provide a new partial characterization result (theorem 1) which gives an interesting set of sufficient conditions for

[3] Of course, the data would be compatible with the maximization of 'menu-dependent' preferences (i.e. dependent on prices), but then *any* observed behavior could be rationalized in this way. This is an approach different from ours, and more in line with, for example Kalai *et al.* [11]. We adhere to the standard revealed preference methodology, in the sense that we assume that consumers use procedures which, though possibly not fully rational, can be falsified by the data.

the demand data to be consistent with the maximization of a strict binary relation. In this set, the main condition simply says that each consumption bundle is observed to be demanded (if at all) at only one price vector (though demand is allowed to be multivalued). Two additional well-behavedness conditions complete the set.

The procedure in (2) is inspired by several sequential eliminative heuristics promoted, for general choice, by prominent psychologists (see e.g. Tversky [20] and Gigerenzer and Todd [6]). Sequential procedures are also specifically documented in the field of consumer choice by marketing scientists: for example Yee et al. [23] offer strong evidence of the use by consumers of "two stage consideration and choice" decision making procedures. Finally, particular versions of such two-stage procedures may look natural when the decision unit that plans consumption is a *collective*, notably a household, rather than a single individual: for example, it looks plausible that first the household members select the Pareto optimal bundles on the basis of their individual preferences - as first proposed by Chiappori [4] - and then they select among the Pareto optimal bundles by means of some 'collective' criterion[4]. This is a more radical departure from ordinary maximization, and it is not obvious which observable conditions on demand data imply, or are implied by, these procedures. In fact, no such conditions exist at the moment, and only indirect estimation algorithms are used to infer boundedly rational procedures[5]. What we have at the moment is only some results in the context of abstract choice from *finite* sets, and therefore not directly applicable to consumer theory). More specifically, in Manzini and Mariotti [14] we study a procedure similar to (2) for single-valued choice functions, while Rubinstein and Salant [17] provide further analysis of our procedure in the finite context[6].

In this paper we show that weakening the sufficient conditions for type (1) of bounded rationality (by dropping the well-behavedness conditions) yields exactly the procedure in (2) (theorem 2). More importantly, we provide a complete characterization (theorem 4). In this characterization, the necessary and sufficient conditions are analogous to the Weak Axiom of Revealed Preference (WARP), in the sense that the asymmetry of a certain type of revealed preference relation, expressed in terms of demand data, is required.

The paper is structured as follows. In the next section we introduce some notation and provide an extended example. In section 3 we carry out the formal analysis. In the concluding section we briefly discuss the results.

2 Notation and an example

2.1 Notation

Let $P \subseteq \mathscr{R}^N_{++}$ denote a set of price vectors. P is interpreted as the set of price vectors at which observations of demanded quantities are made, and it can be finite or (as a limiting case) infinite. Let $X \subseteq \mathscr{R}^N_{+}$ denote a set of consumption bundles. For any $p \in P$, let the competitive budget set $B(p)$ (henceforth simply *budget*) be defined by $B(p) = \{x \in X | px \leq 1\}$.

[4] Interestingly, beyond the sphere of consumer economics, sequential procedures are also used in many other contexts such as recruitement (where shortlisting candidates is a common procedure) and clinical medicine. For example, the online self-help guide of the UK National Health Service (http://www.nhsdirect.nhs.uk/SelfHelp/symptoms/) helps users to recognize an ailment by giving binary answers along a 'tree' of symptoms. This is very different from the possible alternative approach of constructing an aggregate 'index' of symptoms, and may formalize the mental process of a trained doctor.

[5] For recent examples see e.g. Kohli and Jedidi [13] and Yee et al. [23].

[6] We discuss these issues further in the concluding section.

A *demand* is a nonempty correspondence $D : P \to X$ with $D(p) \subseteq B(p)$ for all $p \in P$. It is the map that associates the observed consumption bundles to each observed price (we allow more than one consumption bundle to be observed at each price; the results would hold equally well with single-valued observations).

Vector notation: $x > y$ if $x_i \geq y_i$ for all i and $x \neq y$.

2.2 An Example

The following extended example introduces informally the two boundedly rational procedures we study in this paper.

The frugal consumer. Consider the following demand of the N goods $z \in \mathscr{R}^N_{++}$.

$$D(p) = \arg\min_{pz=1} V(z)$$

where $V : \mathscr{R}^N_+ \to \mathscr{R}$ is a strictly increasing function. The frugal consumer adheres to a simply expressed rule of behavior: she chooses, in each budget, the bundles which, among the efficient ones, minimize the 'aggregate amount' of commodities, where the aggregate amount of commodities is measured by the function V.

How could this observed behavior be modeled in terms of maximization of binary relations? Obviously the answer depends on the function V. Consider first:

$$V(z) = \sum_{i=1}^{N} z_i^2$$

With V thus specified, D is single valued and (as is easy to check) it violates Samuelson's WARP, which say that (with single-valued demand observations) if there is $p \in P$ for which $x \in D(p)$ and y is affordable at p then there is no $p' \in P$ for which $y \in D(p')$ and x is affordable at p'. Therefore D is incompatible with a utility maximization model.

Nonetheless, the frugal consumer's behavior can be expressed in terms of a binary relation K (which, lacking the properties of a preference, we call a 'criterion'). Define, for all $x, y \in X$ with $x \neq y$

$$xKy \Leftrightarrow \nexists p \in P \text{ such that } x \in B(p) \text{ and } y = \arg\min_{pz=1} V(z)$$

Then observed demand is compatible with the following:

d-procedure: Discard, from each budget, all and only those bundles that are worse, according to the criterion K, than some other affordable bundle.

For instance, suppose you only observe the demand from two bundles B_x and B_y as depicted in figure 2.2, where B_i denotes a budget from which bundle $i = x, y$ is demanded. It is easy to see that xKz for all bundles $z \in B_x$ such that $z \neq y$, since with the two observations you have there are no price vectors such that $z \neq y$ solves the constrained minimization problem when x is affordable, the only exception being bundle y. But, for a bundle like x' in figure 2.2 it is $x'Ky$, since there are no (observed) budgets such that x' is affordable when y minimizes the objective function. Consequently, the d-procedure selects x uniquely from budget B_x (and similarly one can verify that it selects y uniquely from budget B_y). Note in

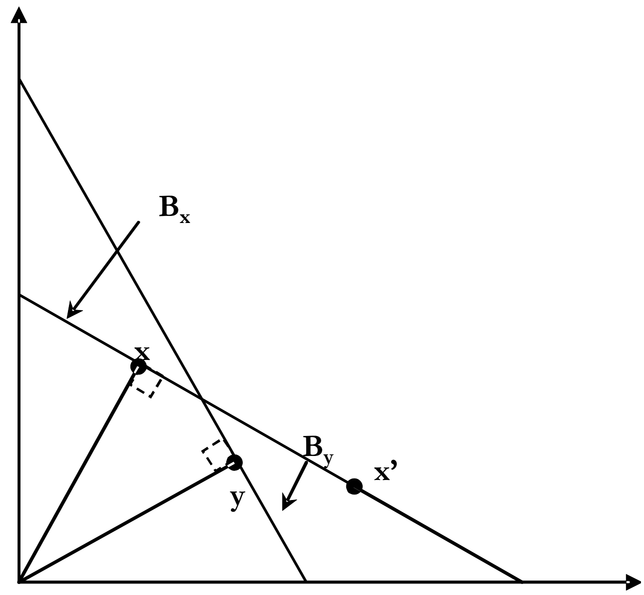

Fig. 2 The frugal consumer aggregates with $\sum_i z_i^2$

particular that, unlike the case of utility maximization, y is 'eliminated' not by the demanded bundle x itself, but rather by bundles that are themselves not demanded.

Next, consider the following alternative specification for the aggregation function:

$$V(z) = \sum_{i=1}^{N} z_i$$

The resulting (multivalued) demand is no longer explainable via the d-procedure. To see this, let $N = 2$ and consider prices $p = (1,1)$. The entire budget line is the demand at these prices. So, suppose there is some relation K that 'rationalizes' demand. Letting $x = (1,0)$, we have yKx for no y affordable at p. Then at prices $p' = (1,2)$, x is still affordable, and it should be chosen since $B(p') \subset B(p)$ and therefore yKx for no y affordable at p'. But it is easy to check that $D(p') = \left\{\left(0, \frac{1}{2}\right)\right\}$ (see figure 3).[7]

However, demand may be recovered by *sequentially* applying *two* relations K_1 and K_2 defined as follows:

$$xK_1y \text{ if and only if } x > y$$
$$xK_2y \text{ if and only if } \sum_{i=1}^{N} x_i < \sum_{i=1}^{N} y_i$$

These can be used in the following alternative procedure, which extends the previous one by adding a round of 'selection':

[7] Formally, this demand function violates the V-axiom defined in the next section.

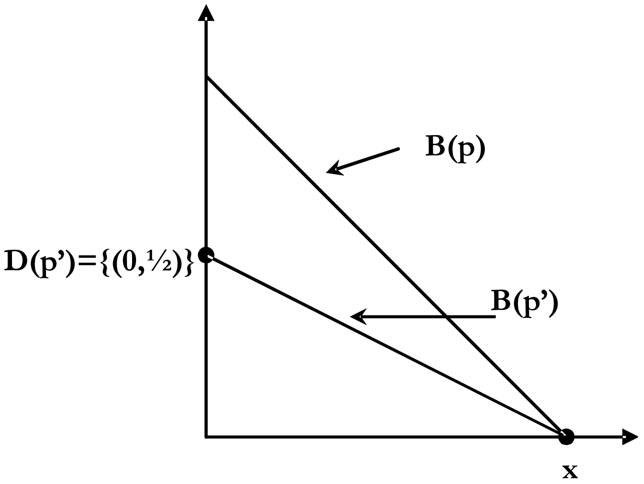

Fig. 3 The frugal consumer aggregates with $\sum_i z_i$

dc-procedure First *discard* from each budget all and only those bundles that are worse than some other affordable bundle according to K_1; then, among the surviving bundles, *choose* all and only those bundles that according to K_2 are not worse than any other surviving bundle and are better than each surviving rejected one.

Note in particular that with this procedure, in the second round any bundle which is rejected is eliminated by the bundles which are actually demanded. In this sense the term 'choose', rather than 'discard' appears justified.

In the example, with the dc-procedure, first all the Pareto dominated bundles are discarded from a budget, and on the budget line the 'Utilitarian' minimizers are selected, thus yielding exactly the demand D.

3 Boundedly rational procedures

In this section we define formally the two procedures illustrated with the frugal consumer example. The following concept originated in Kim and Richter [12]).[8]

A demand D is **discard rational (d-rational)** if there exists a relation K such that

$$D(p) = \{x \in B(p) \,|\, yKx \text{ for no } y \in B(p)\}$$

In this case we say that K *d-rationalizes*, or is a *d-rationale for*, the demand.[9]

Next we introduce a two-stage boundedly rational procedure. Given a binary relation K on $\mathscr{R}_+^N$ and $S \subset \mathscr{R}_+^N$, denote $\max(S,K) = \{x \in S | yKx \text{ for no } y \in S\}$.

[8] They use the term 'motivated' instead of d-rational.

[9] In their elegant paper Kim and Richter [12] show that a demand D is d-rational if and only if there exists a relation K such that

$$D(p) = \{x \in B(p) \,|\, xKy \text{ for all } y \in B(p)\}$$

The demand from each budget is (weakly) 'better' than all elements in the budget according to the relation K.

A demand D is **discard and choose rational (dc-rational)** if there exist two relations K_1 and K_2 such that

$$D(p) = \max(\max(B(p), K_1), K_2)$$
$$\text{and } xK_2y \text{ for all } x \in D(p) \text{ and } y \in \max(B(p), K_1) \setminus D(p)$$

In this case we say that K_1 and K_2 *dc-rationalize*, or are *dc-rationales for*, the choice.[10]

Observe that, crucially, the two dc-rationales are always applied in the same order for all budgets.

3.1 Sufficient conditions

We provide partial characterizations of the above concepts in terms of the following conditions on demand, which are very easy to observe.

D is *inverse single-valued* iff $D^{-1}(x)$ is empty or single valued for all $x \in X$.

D is *interior* iff $x \in \mathscr{R}^N_{++}$ for all $x \in D(p)$, for all $p \in P$.

D is *budget exhaustive* iff $px = 1$ for all $x \in D(p)$, for all $p \in P$.

In the case of ordinary utility maximization, all these properties are satisfied, for example, by the Cobb-Douglas demand function.

Theorem 1 *Any inverse single-valued, interior and budget-exhaustive demand is d-rational.*

Proof: Suppose that D satisfies the properties and define the d-rationale K as follows: xKy iff $y \notin D(p)$ for all $p \in P$ such that $x, y \in B(p)$.

Consider any $p \in P$ and let $x \in D(p)$. Then for no $y \in B(p)$ can it be the case that yKx. Let $y \notin D(p)$. We show that zKy for some $z \in B(p)$. Suppose not. Then for all $z \in B(p)$ there exists $p_z \in P$ such that $y \in D(p_z)$ and $z \in B(p_z)$. By inverse single-valuedness it must be that there exists some $p_y \in P$ such that $p_z = p_y$ for all p_z. In particular, this implies $B(p) \subseteq B(p_y)$, and $y \in \mathscr{R}^N_{++}$ by interiority. We show that this implies a contradiction.

Since $y \in B(p)$, we have $py \leq 1 = p_y y$ (where the equality follows from budget exhaustion). Consider first the case $py = 1 = p_y y$. So (given that $y \in \mathscr{R}^N_{++}$) either $p = p_y$ or there exists i, j with $p_i > (p_y)_i$ and $p_j < (p_y)_j$. The former case cannot apply, for then $D(p) = D(p_y)$, contradicting the assumptions $y \notin D(p)$ and $y \in D(p_y)$. The latter case cannot apply either. If it did, we could construct a bundle y' with $y' \in B(p)$ and $y' \notin B(p_y)$ by setting, for small $\varepsilon, \eta > 0$, $y'_i = y_i - \varepsilon$ (possible by $y \in \mathscr{R}^N_{++}$), $y'_j = y_j + \eta$, with $\varepsilon p_i = \eta p_j$, and $y_k = y'_k$ for all other k. The existence of such a y' contradicts $B(p) \subseteq B(p_y)$.

It remains to consider the case $py < 1$. This is impossible in view of $B(p) \subseteq B(p_y)$ and budget exhaustion. ■

The next result shows that by dropping the interiority and budget exhaustion conditions we can still model demand behavior in terms of binary relations and bounded rationality.

[10] Alternatively:

$$D(p) = \{x \in \max(B(p), K_1) : xK_2y \text{ for all } y \in \max(B(p), K_1) \setminus D(p)\}$$

Theorem 2 *Any inverse single-valued demand is dc-rational.*

Proof. Let D be inverse single-valued. Define the dc-rationale K_1 by xK_1y iff $y \notin D(p)$ for all $p \in P$ such that $x, y \in B(p)$. Define the dc-rationale K_2 as follows: xK_2y iff there exists $p \in P$ such that $x \in D(p)$, $y \in B(p) \setminus D(p)$ and there exists p_y such that $B(p) \subseteq B(p_y)$ and $y \in D(p_y)$. By inverse single-valuedness p and p_y are unique if they exist.

Consider any $p \in P$ and suppose that $y \in B(p) \setminus D(p)$. We show that either zK_1y for some $z \in B(p)$ or xK_2y for all $x \in D(p)$. Suppose that zK_1y for no $z \in B(p)$. Then there exists a unique (by inverse single-valuedness) $p_y \in P$ such that $y \in D(p_y)$ and $B(p) \subseteq B(p_y)$. By definition this implies that xK_2y for all $x \in D(p)$.

Now take $x \in D(p)$. It is obvious that zK_1x for no $z \in B(p)$. We show that yK_2x for no $y \in \max(B(p), K_1)$. Suppose not, and let yK_2x for some $y \in \max(B(p), K_1)$. Then there exists a unique $p_y \in P$ with $y \in D(p_y)$, $x \in B(p_y) \setminus D(p_y)$ and $B(p_y) \subseteq B(p)$. On the other hand if $y \in \max(B(p), K_1)$, by definition of K_1 it must be $B(p) \subseteq B(p_y)$. Therefore $B(p_y) = B(p)$, contradicting $x \in D(p)$ and $x \notin D(p_y)$. ■

3.2 Characterizations

Next, we move to conditions on observed demand which, though less simple, completely characterize d- and dc-rationality.

The key result on d-rationality, due to Kim and Richter [12], is reported after the following definition[11]:

The V-Axiom. For all $p \in P$ and $x \in B(p)$: if for all $z \in B(p)$ there exists $p_z \in P$ such that $x \in D(p_z)$ and $z \in B(p_z)$, then $x \in D(p)$.

Theorem 3 *(Kim and Richter [12]) A demand is d-rational if and only if it satisfies the V-Axiom.*

The concept of dc-rationality is more permissive than that of d-rationality. As we saw above, there are dc-rational demands that are not d-rational. Are all demands dc-rational? The answer is no, and we provide below two examples illustrating two different ways in which dc-rationality may fail.

A lonely decision maker wants to treat himself on the night of his birthday, purchasing a meal either in restaurant A or in restaurant B. In order to enjoy the 'treat' element he wants to buy the more expensive meal, if affordable. More generally, this example captures the case where a decision maker has some ideal level of consumption of a good of a given type, and his preferences are such that he wishes to purchase exactly that amount of the most expensive type, if affordable. Suppose that in budget $B(p_y)$ a meal in restaurant B is relatively more expensive than a meal in restaurant A, while the opposite is true in budget $B(p_x)$, and in budget $B(p)$ both meals are equally expensive. If we let bundles x and y , indicate the optimal amounts of either only good of type A or only good of type B (e.g. a four course set meal in restaurant A and B, respectively), this criterion generates the following demand:

$$D(p_x) = \{x\}, D(p_y) = \{y\}, D(p) = \{x, y\}$$
$$\text{with } B(p) \subset B(p_y) \subset B(p_x)$$

as depicted in figure 4.

[11] Campbell [3] also reports interesting conditions for d-rationality. However, they require the convexity of P and are therefore not suited to the domain of application we have in mind here.

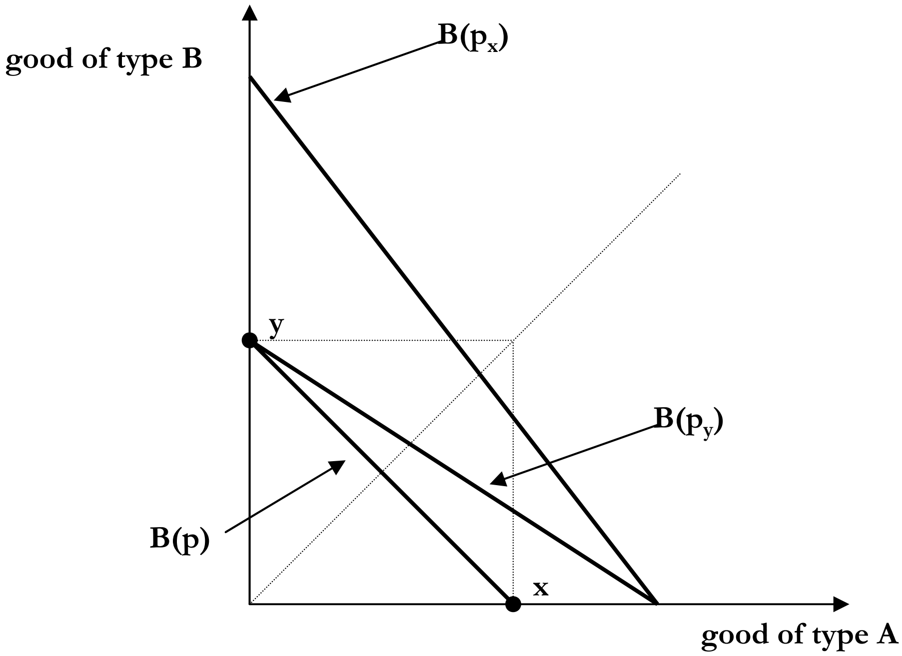

Fig. 4 A lonely soul's treat on his birthday night out

Indeed, this demand, though plausible, is not dc-rational. To see this, observe that since bundle x is rejected from budget $B(p_y)$ while bundle y is demanded, it must be that yK_2x. But this then makes it impossible for x to be maximal with respect to K_2 in budget $B(p)$ (one could have made a symmetric reasoning observing that since bundle y is rejected from budget $B(p_x)$ while bundle x is demanded, it must be that xK_2y).

Consider now the following demand:

$$D(p_2) = \{x\}, D(p_1) = \{y\} = D(p_3)$$
$$\text{with } B(p_1) \subset B(p_2) \subset B(p_3)$$

depicted in figure 5, where as before x and y refer to bundles with the optimal amounts of either only good of type A or only good of type B, respectively. Observe that in both $B(p_1)$ and $B(p_3)$ meal y is relatively more expensive than meal x, while the opposite is true in budget $B(p_2)$.

Once more this demand, though reasonable in the context of our story, is not dc-rational. To see this, observe that since bundle y is rejected from budget $B(p_2)$ while bundle x is demanded, it must be that xK_2y. But this then makes it impossible for y to be maximal with respect to K_2 in budget $B(p_1) \subset B(p_2)$.[12]

In order to provide a characterization of dc-rationality in terms of observable demand behavior, we introduce a new type of revealed preference relation. Denote the standard direct revealed preference relation (x is demanded and y is rejected from some budget) by P_D: that

[12] Examples similar to those presented can easily be found even under the assumption that demand is budget-exhaustive, though not with only two goods.

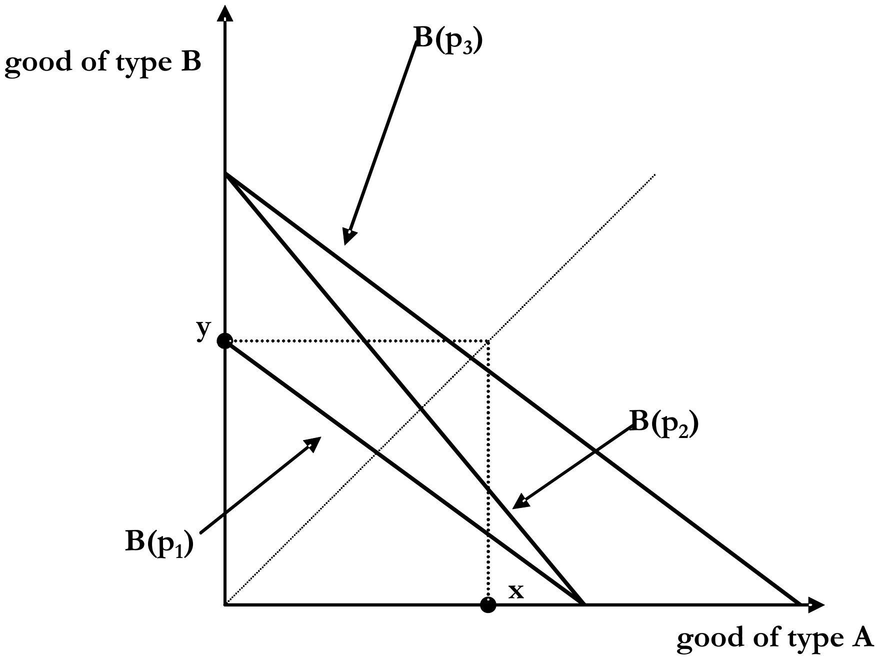

Fig. 5 Another birthday

is, xP_Dy if and only if there exists $p \in P$ such that $x \in D(p)$ and $y \in B(p) \setminus D(p)$. Samuelson proposed the asymmetry of the direct revealed preference relation as the observable test of utility maximization[13]. We introduce a similar asymmetry test, only applied to a weaker revealed preference relation, P_D^*. We say that xP_D^*y if and only if there exists $p \in P$ such that $x \in D(p)$ and $y \in B(p) \setminus D(p)$ (that is xP_Dy) and for all $z \in B(p)$ there exists p_z such that $y \in D(p_z)$ and $z \in B(p_z)$. That is, x is directly revealed preferred to y in a budget, but at the same time y is demanded in the presence of any alternative bundle in that budget.

Axiom A: $[xP_D^*y] \Rightarrow$ *not* $[yP_D^*x]$.

Axiom B: for all $p \in P$: $[x, y \in D(p)] \Rightarrow$ *not* $[yP_D^*x]$.

Remark 1 For single-valued demand (the data are single choice observations from budgets) Axiom B is vacuously satisfied[14].

Remark 2 The two axioms are independent, as demonstrated by the two examples above. In the example of figure 4, note that $D(p_x) = \{x\}$, $D(p_y) = \{y\}$ and $B(p_y) \subset B(p_x)$ imply that yP_D^*x while *not* (xP_D^*y), so that Axiom A holds. However Axiom B fails since yP_D^*x and $D(p) = \{x, y\}$.

In the example of figure 5, note that $D(p_2) = \{x\}$, $D(p_1) = \{y\}$ and $B(p_1) \subset B(p_2)$ imply that xP_D^*y, while $D(p_2) = \{x\}$, $D(p_3) = \{y\}$ and $B(p_2) \subset B(p_3)$ imply that yP_D^*x, so that Axiom A fails. Axiom B holds trivially, as the observed demand is single valued.

[13] However, this in fact only works for two commodities. The asymmetry of P_D, or WARP, does not characterize in general any standard notion of rational behavior, though Kim and Richter [12] do provide a partial characterization, and Clarke [5] and Mariotti [15] relate it to a modified notion of rationality.

[14] Note that if demand is single valued, the premise of Axiom B is always false.

Our main characterization result is the following:

Theorem 4 *A demand is dc-rational if and only if it satisfies Axiom A and Axiom B.*

Proof. Let D be dc-rationalized by K_1 and K_2. Suppose that xP_D^*y, and in particular that (i) $x \in D(p)$, (ii) $y \in B(p) \setminus D(p)$, and (iii) for all $z \in B(p)$ there exists p_z such that $y \in D(p_z)$ and $z \in B(p_z)$. Condition (iii) implies that there is no $z \in B(p)$ for which zK_1y.

We show that P_D^* is asymmetric (Axiom A). Conditions (i), (ii) and (iii) and the dc-rationality of D imply that xK_2y. Now take any p' such that $y \in D(p')$ and $x \in B(p') \setminus D(p')$ (if no such p' exists, then the asymmetry of P_D^* follows vacuously). For D to be dc-rational, it must be the case that x is eliminated in the first stage, that is there exists $z \in B(p')$ such that zK_1x. Therefore it cannot be the case that for all $z \in B(p')$ there exists p_z such that $x \in D(p_z)$ and $z \in B(p_z)$, so it cannot be yP_D^*x.

Next we show that for all $p' \in P$, *not* $[x, y \in D(p')]$ (Axiom B). Since xP_D^*y, it must be the case for D to be dc-rational that xK_2y. Then it cannot be the case that $x, y \in D(p')$ with D dc-rational.

For the other direction of the statement, let D satisfy the axioms, and define K_1 as xK_1y iff $y \notin D(p)$ for all $p \in P$ such that $x, y \in B(p)$. Define $K_2 = P_D^*$.

Consider any $p \in P$ and suppose that $y \notin D(p)$. We show that either zK_1y for some $z \in B(p)$ or xK_2y for all $x \in D(p)$. Suppose that zK_1y for no $z \in B(p)$. Then for all $z \in B(p)$ there exists p_z such that $z \in B(p_z)$ and $y \in D(p_z)$. By definition this means that xK_2y for any $x \in D(p)$.

Now take $x \in D(p)$. It is obvious that zK_1x for no $z \in B(p)$. We show that yK_2x for no $y \in \max(B(p), K_1)$. Suppose not, and let yK_2x. Then by Axiom A K_2 $(= P_D^*)$ is asymmetric, so it must be $y \in D(p)$ (otherwise, since $x \in D(p)$ and y survives the first round, we would have also xK_2y). But this contradicts Axiom B. ■

Compared with the standard SARP and GARP conditions on demand[15], our key Axiom A is simpler in one respect and more complex in another. It is simpler, because it uses a direct revealed preference relation (P_D^*) instead of the transitive closure of a direct relation. However, more observations are needed to check xP_D^*y than to check xP_Dy. With two commodities, for example, we need at least two observations (one to check that x is chosen from a budget $B(p)$, and at least another one to check that y is demanded in the presence of any alternative bundle in $B(p)$). If budget exhaustion were required, then unless y is in a corner, one needs at least three observations (at least two to check that y is demanded in the presence of any alternative bundle in $B(p)$).

Also, observe that when the direct revealed preference relation P_D is asymmetric as proposed by Samuelson, the relation P_D^* is empty. In this case, it is the first stage only that governs behavior.

Another natural procedure related to dc-rationality is expressed as follows. A demand D is **dd-rational** if there exist two relations K_1 and K_2 such that

$$D(p) = \max(\max(B(p), K_1), K_2)$$

The definition is the same as dc-rationality except for the fact that in this case the demanded bundles do not need to dominate the discarded ones at the second stage. A revealed preference characterization of dd-rationality is still an open question. This is the exact analogue in a consumer theory setting of the notion of Rational Shortlist Methods (RSM),

[15] Let the demand D be single-valued. Let P_D^t be the transitive closure of P_D. D satisfies SARP if P_D is acyclic (or equivalently P_D^t is asymmetric). D satisfies GARP if xP_D^ty implies $px \geq py$ for all $p \in P$ such that $y = D(p)$.

which, as we mention in the introduction, we introduced and characterized in Manzini and Mariotti [14]. A choice function γ is an RSM whenever there exist two asymmetric binary relations K_1 and K_2 such that, for every feasible set S, $\gamma(S) = \max(\max(S, K_1), K_2)$. Interestingly, our characterization of (single-valued) RSMs relies on axioms that cannot be extended trivially to a consumer setting. More precisely, RSMs are characterized (on the set of all subsets of a finite set) by a weakening of WARP and by an expansion axiom. As for the latter, we require that if the same element is chosen from two distinct menus, then this same element should be chosen from the union of the two menus. Such an axiom would not serve any purpose in a consumer theory setting, given the lack of closure of the set of budget sets under set union. Similarly, the weakening of WARP that we employ in the finite domain imposes conditions on choices from binary sets, which of course are not the object of analysis in consumer theory[16].

4 Concluding remarks

We have proposed characterizations of two boundedly rational procedures in consumer choice. The conditions in these characterizations are expressed only in terms of observable demand data, and therefore are of the same nature as the conditions of ordinary revealed preference theory. They allow nonparametric tests of the decision procedures we have studied, just as GARP allows nonparametric tests of the model of consumer utility maximization.

To labor this point, note that we have not expanded the domain of choice to include unusual data, as is sometimes done in the search for testable conditions on models explaining behavioral 'anomalies'. As a notable example, in order to explain time inconsistent choices between dated alternatives, Gul and Pesendorfer in a remarkable series of papers[17] enlarge the domain of preferences to include, beside standard alternatives, also 'menus' of alternatives. While theoretically elegant, this approach uses data (such as preferences over menus) which are not part of standard consumer microeconometrics data (e.g. family budget surveys)[18]. So, although that approach leads to conditions which are testable *in principle*, such conditions are one step removed from the ordinary revealed preference data for consumer choice. It is certainly possible to infer consumers' preferences over *some* menus (for example their rejection of some alternatives through commitment may be construed as their choice of a menu), but obtaining the entire preference relation needed for the theory may be a tall order, and will certainly need additional data and empirical techniques. For this reason, that approach may be more suitable for experimental evidence rather than market data.

In a recent paper, Rubinstein and Salant [17] propose a model in which choice data are, as standard, subsets of the (finite) sets of feasible alternatives. However, the choice is made to depend not only on the feasible set itself, but also on another object, called a *frame*. In other words, the choice from a set S is formalized as $c(S, f)$, where f is the frame. An example of a frame is the order in which the alternatives in S are presented. Another example is a distinguished alternative (e.g. the status quo) in S. Frames are conceived as observable objects, so this is yet another method for enriching the set of data available in a revealed preference analysis.

We view our approach in this paper as complementary to these contributions, in the sense that it is entirely feasible with the standard techniques and data applied to consumer choice.

[16] Weak WARP: If $y \neq x = \gamma(\{x, y\}) = \gamma(T)$ and $\{x, y\} \subseteq S \subseteq T$ then $y \neq \gamma(S)$.

[17] E.g. Gul and Pesendorfer [7], [8], [9].

[18] See e.g. Blundell [2].

Acknowledgements We thank two anonymous referees and a co-Editor of this Journal, and Stepana Lazarova, Bart Lipman and Chris Tyson for useful suggestions. The responsibility for errors is, as usual, ours.

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